The New Keynesian Model Part 2

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Macroeconomics II

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Calvo pricing with interest rate targeting

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- In our model this far, the nominal interest rate is a constant.
- Yet, in the data, it moves around, and we have seen that this potentially matters for output fluctuations.
- We could get some volatility by having $\rho_m > 0$ or assuming a money demand function that is not in logs.
- However, the major conceptual shortcoming is probably to assume a stable money demand function.

- A way to overcome this problem is to focus on the price of the money market instead of quantities.
- The nice thing is that central banks tell us that this is what they are doing, i.e., they target an interest rate.
- That is, they adjust the money supply until they reach the desired price.
- The nice thing is that we can interpret shocks to the system $S_t = f(\Omega_t) + \epsilon_t$ to come either from the demand or supply side and forget about the details of the money market equilibrium (i.e., the LM-curve).

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A monetary policy rule

- The question is, what does f(Ω_t) look like?
- Taylor (1993) shows that the actual policy is well-approximated by the following simple rule:

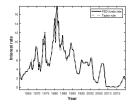
$$i_{t} = i^{ss} + \kappa_{\pi}(\pi_{t} - \pi^{ss}) + \kappa_{y}(\hat{Y}_{t} - \hat{Y}_{t}^{f}) + \omega_{t}$$
(1)
$$\omega_{t} = \rho_{i}\omega_{t-1} + \epsilon_{t}^{i}$$
(2)

- That is, the nominal interest rate is a constant plus a policy term.
- The policy term says that the central bank raises the interest rate when the output gap is positive and when inflation is above its long-run target.
- Errors are a persistent AR(1) process.



- The Taylor rule matches many dynamics from the data.
- The main disagreement is after the Great Recession. CBO has calculated that the potential output has been reached, yet, the FED has kept the interest rate at zero.

The Taylor rule II



• Later studies suggest that letting the interest target follow an AR(1) process yields a yet better fit:

$$\dot{h}_t = (1-
ho_i)\dot{h}^{ss} + (1-
ho_i)\left[\kappa_\pi(\pi_t-\pi^{ss})+\kappa_y(\hat{Y}_t-\hat{Y}_t^f)
ight] +
ho_i\dot{h}_{t-1} + \epsilon_t^i$$

 For example, the central bank could be concerned about creating a stable environment for banks.

- The model is the same as before but instead of a money growth rule we now have the Taylor rule.
- This makes it unnecessary to think about money demand, i.e., we can forget about money in the utility function and the corresponding money demand equation.
- You can think about a money market being active in the background, and the central bank adjusts its money supply to achieve the intended interest rate.
- However, in doing so, it makes persistent mistakes, ϵ_t^i .

- Note, in the presence of sticky prices, the central bank affects the economy in two ways. First, through predictable changes in the nominal interest rate and second through unpredictable changes.
- Regarding predictable changes, note the difference to the money growth rate rule: When the output gap becomes negative, the central bank decreases the interest rate. Hence, after a rise in productivity which leads to a negative output gap, the central bank increases the boom by decreasing the interest rate (increase the money supply). Put differently, the central bank overcomes partially the price stickiness that lead to inefficient business cycle volatility.

Household's FOCs:

$$C_{t}^{-\gamma} = \beta \mathbb{E}_{t} \left\{ C_{t+1}^{-\gamma} (1+i_{t})(1+\pi_{t+1})^{-1} \right\}$$
(3)
$$\phi H_{t}^{\eta} = C_{t}^{-\gamma} \frac{W_{t}}{P_{t}}.$$
(4)

Budget constraint and output:

$$Y_t = C_t \tag{5}$$

$$Y_t = \frac{A_t H_t}{\psi_t} \tag{6}$$

$$\psi_t = (1 - \lambda) \left(1 + \pi_t^{\circ} \right)^{-\mu} \left(1 + \pi_t \right)^{\mu} + \lambda \psi_{t-1} \left(1 + \pi_t \right)^{\mu}.$$
 (7)

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Firm optimality:

$$(1 + \pi_t)^{1-\mu} = (1 - \lambda)(1 + \pi_t^{\circ})^{1-\mu} + \lambda$$
(8)

$$(1 + \pi_t^{\circ}) = \frac{\mu}{\mu - 1} (1 + \pi_t) \frac{x_{1,t}}{x_{2,t}}$$
(9)

$$x_{1,t} = C_t^{-\gamma} \frac{W_t}{A_t P_t} Y_t + \beta \lambda \mathbb{E}_t x_{1,t+1} (1 + \pi_{t+1})^{\mu}$$
(10)

$$x_{2,t} = C_t^{-\gamma} Y_t + \beta \lambda \mathbb{E}_t x_{2,t+1} (1 + \pi_{t+1})^{\mu - 1}.$$
 (11)

Exogenous processes:

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1} \tag{12}$$

$$\dot{i}_t = \dot{i}^{ss} + \kappa_\pi (\pi_t - \pi^{ss}) + \kappa_y (\hat{Y}_t - \hat{Y}_t^f) + \omega_t$$
(13)

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Log-linearizing the Taylor rule is straight forward:

$$i_t = i^{ss} + \kappa_\pi (\pi_t - \pi^{ss}) + \kappa_y (\hat{Y}_t - \hat{Y}_t^f) + \omega_t$$
(14)

$$\dot{i}_t - \dot{i}^{ss} = \kappa_\pi (\pi_t - \pi^{ss}) + \kappa_y (\hat{Y}_t - \hat{Y}_t^f) + \omega_t$$
(15)

Deviations of the interest rate from its steady state depend on inflation deviations from steady state, output deviations from the flexible price equilibrium, and shocks to the monetary policy.

- We have a system of log-linear equations that we can solve.
- It is common to further reduce the system of log-linear equations to just 4 equations.
- This system will be only defined in terms of the output gap deviations from steady state, not output deviations from steady state.
- Also, we have to introduce a new variable: the natural real rate of interest.

We start by substituting our \hat{A}_t from the equations using

$$\hat{Y}_t^f = \frac{1+\eta}{\gamma+\eta} \hat{A}_t \tag{16}$$

$$=\frac{1+\eta}{\gamma+\eta}[\rho\hat{A}_{t-1}+\epsilon_t]$$
(17)

$$=\frac{1+\eta}{\gamma+\eta}[\rho\frac{\gamma+\eta}{1+\eta}\hat{Y}_{t-1}^{f}+\epsilon_{t}]$$
(18)

$$=\rho\hat{Y}_{t-1}^{f} + \frac{1+\eta}{\gamma+\eta}\epsilon_t.$$
(19)

The flexible price equilibrium simply follows an AR(1) process. This comes from the fact that we have no capital as state variable and, hence, the only relevant state is productivity which follows an AR(1) process. Next, we write the IS-curve in terms of the output gap:

$$\mathbb{E}_{t} \hat{Y}_{t+1} - \hat{Y}_{t} = \frac{1}{\gamma} \left[(i_{t} + \ln(\beta)) - \mathbb{E}_{t} \pi_{t+1} \right]$$

$$\mathbb{E}_{t} (\hat{Y}_{t+1} - \hat{Y}_{t+1}^{f}) - \hat{Y}_{t}^{f} = \hat{Y}_{t} - \hat{Y}_{t}^{f} - \mathbb{E}_{t} \hat{Y}_{t+1}^{f} + \frac{1}{\gamma} \left[(i_{t} + \ln(\beta)) - \mathbb{E}_{t} \pi_{t+1} \right]$$

$$(21)$$

$$\hat{Y}_{t} - \hat{Y}_{t}^{f} = \mathbb{E}_{t} (\hat{Y}_{t+1} - \hat{Y}_{t+1}^{f}) + \mathbb{E}_{t} \hat{Y}_{t+1}^{f} - \hat{Y}_{t}^{f} - \frac{1}{\gamma} \left[(i_{t} + \ln(\beta)) - \mathbb{E}_{t} \pi_{t+1} \right]$$

(22)

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Define the natural real rate of interest as deviation from its steady state, $\tilde{r}_t^f = r_t^f + \ln(\beta)$, as the interest rate that prevails when prices are flexible, $\lambda = 0$. In that case, we know that $\hat{Y}_t = \hat{Y}_t^f$

$$0 = 0 + \mathbb{E}_t \hat{Y}_{t+1}^f - \hat{Y}_t^f - \frac{1}{\gamma} (r_t^f + \ln(\beta))$$
(23)

$$\tilde{r}_t^f = (r_t^f + \ln(\beta)) = \gamma(\mathbb{E}_t \hat{Y}_{t+1}^f - \hat{Y}_t^f).$$
(24)

The deviation of the natural real rate of interest from its steady state is simply proportional to the expected growth rate of flexible price output from its steady state. The intuition is simple: In the flexible price equilibrium, for households to be willing to accept output (consumption) growth, they need to be compensated by a higher real interest rate.

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Plugging this result into the IS-curve eliminates the deviations of flexible price output from its steady state:

$$\mathbb{E}_t(\hat{Y}_{t+1} - \hat{Y}_{t+1}^f) - (\hat{Y}_t - \hat{Y}_t^f) = \frac{1}{\gamma} \left[(i_t + \ln(\beta)) - \mathbb{E}_t \pi_{t+1} - \tilde{r}_t^f \right] \quad (25)$$

The expected growth in the output gap (in deviations from steady state) depends negatively on the deviation of today's real interest rate from its flexible price equilibrium (in deviations from steady state). Put differently, a too high real interest rate today causes households to defer consumption causing a negative output gap today.

A process for the natural real rate of interest

Note, using (19), that $\mathbb{E}_t \hat{Y}^f_{t+1} = \rho \hat{Y}^f_t$. Hence,

$$\tilde{r}_t^f = \gamma(\mathbb{E}_t \hat{Y}_{t+1}^f - \hat{Y}_t^f)$$
(26)

$$=\gamma(\rho-1)\hat{Y}_t^f \tag{27}$$

$$=\gamma(\rho-1)[\rho\hat{Y}_{t-1}^{f}+\frac{1+\eta}{\gamma+\eta}\epsilon_{t}]$$
(28)

$$= \gamma(\rho - 1) \left[\rho \frac{1}{\gamma(\rho - 1)} \tilde{r}_{t-1}^{f} + \frac{1 + \eta}{\gamma + \eta} \epsilon_{t}\right]$$
(29)
$$= \rho \tilde{r}_{t-1}^{f} + \gamma(\rho - 1) \frac{1 + \eta}{\gamma + \eta} \epsilon_{t}.$$
(30)

Deviations of the natural real rate of interest from steady state are a simple AR(1) process.

At the zero inflation steady state, the system of equilibrium conditions is simply:

$$\mathbb{E}_{t}(\hat{Y}_{t+1} - \hat{Y}_{t+1}^{f}) - (\hat{Y}_{t} - \hat{Y}_{t}^{f}) = \frac{1}{\gamma} \left[(i_{t} - i^{ss}) - \mathbb{E}_{t} \pi_{t+1} - \tilde{r}_{t}^{f} \right]$$
(31)

$$i_t - i^{ss} = \kappa_\pi \pi_t + \kappa_y (\hat{Y}_t - \hat{Y}_t^f) + \omega_t$$
(32)

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} (\eta+\gamma) [\hat{Y}_t - \hat{Y}_t^f] + \beta \mathbb{E}_t \pi_{t+1}$$
(33)

$$\tilde{r}_t^f = \rho \tilde{r}_{t-1}^f + \gamma (\rho - 1) \frac{1 + \eta}{\gamma + \eta} \epsilon_t,$$
(34)

which are four equations in four unknowns (output gap, interest gap, inflation, and the real natural rate gap). This system is sometimes called the 3-equation New Keynesian model.

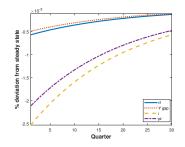
Calibration

- We are going to use the same calibration as before with an inverse labor supply elasticity $\eta = 0.5$ This leads to $\phi = 12$ to match $H^{ss} = 0.33$.
- We will look at the non-inflationary steady-state.
- In the data, the average price duration is 13 months in Europe. In the model, average duration in quarters is $\frac{1}{1-\lambda}$. Hence, $\lambda = 0.75$ is reasonable.
- Estimating the Taylor rule yields $\kappa_y = 0.2$ and $\kappa_{\pi} = 1.15$, i.e., the central bank responds stronger to inflation that to output deviations.
- Finally, I target with the shocks to the interest rate a standard deviation in the nominal rate of 1.07% which yields $\sigma_i = 0.014$ and with ρ_i an autocorrelation in the error of the Taylor rule of 0.2.

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- We are going to start with the 3-equation NKM.
- In that version, the mechanisms are very transparent.
- However, it is only informative about the output gap and hours are no longer part of the solution.
- As we are interested in the volatility of output and hours, we turn back to the full model thereafter.

A positive productivity shock



• A positive productivity shock leads, relative to the steady state, to

- A decrease in the real natural rate of interest.
- A negative output gap.
- Decreases in the nominal interest and inflation rates.

The law of motion for the real natural rate

$$\tilde{r}_t^f = \rho \tilde{r}_{t-1}^f + \gamma (\rho - 1) \frac{1 + \eta}{\gamma + \eta} \epsilon_t$$

directly tells us that the deviation of the real natural rate of interest from steady state becomes negative. Hence, from the IS-curve:

$$\mathbb{E}_t(\hat{Y}_{t+1} - \hat{Y}_{t+1}^f) - (\hat{Y}_t - \hat{Y}_t^f) = \frac{1}{\gamma} \left[(i_t - i^{ss}) - \mathbb{E}_t \pi_{t+1} - \tilde{r}_t^f \right]$$

we know that we get an expected growth in the output gap which is achieved by a fall in the output gap today, i.e., demand is insufficient to cover the potential supply. The central bank reacts to the fall in the output gap by

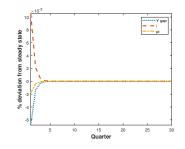
$$\dot{i}_t - i^{ss} = \kappa_\pi \pi_t + \kappa_y (\hat{Y}_t - \hat{Y}_t^f) + \omega_t$$

decreasing the nominal interest rate. Moreover, firms react to a negative output gap by

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}(\eta+\gamma)[\hat{Y}_t - \hat{Y}_t^f] + \beta \mathbb{E}_t \pi_{t+1}$$

decreasing prices.

A positive interest shock



• A positive interest rate shock leads, relative to the steady state, to

- An increase in the nominal interest rate.
- A negative output gap.
- A decrease in the inflation rate.

The central bank causes the nominal interest rate to rise.

$$i_t - i^{ss} = \kappa_\pi \pi_t + \kappa_y (\hat{Y}_t - \hat{Y}_t^f) + \omega_t$$
$$\omega_t = \rho_i \omega_{t-1} + \epsilon_t^i.$$

Hence, from the IS-curve:

$$\mathbb{E}_t(\hat{Y}_{t+1} - \hat{Y}_{t+1}^f) - (\hat{Y}_t - \hat{Y}_t^f) = \frac{1}{\gamma} \left[(i_t - i^{ss}) - \mathbb{E}_t \pi_{t+1} - \tilde{r}_t^f \right]$$

we know that we get an expected growth in the output gap which is achieved by a fall in the output gap today, i.e., demand is insufficient to cover the potential supply. Firms react to a negative output gap by

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}(\eta+\gamma)[\hat{Y}_t - \hat{Y}_t^f] + \beta \mathbb{E}_t \pi_{t+1}$$

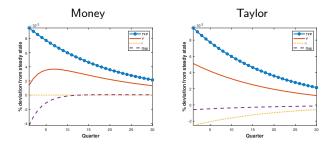
decreasing prices. Note, the law of motion for the real natural rate, the only backward looking variable,

$$ilde{r}_t^f =
ho ilde{r}_{t-1}^f + \gamma (
ho - 1) rac{1+\eta}{\gamma+\eta} \epsilon_t$$

is not affected. Hence, the shock has only persistent effects because the central bank makes persistent mistakes (or smooths interest rates).

- We are now going to study the quantitative results in the full model.
- This allows us to study output, not only the output gap.
- This allows us to study the real interest rate, not only the real interest rate gap.
- This allows us to study wages.
- This allows us to study hours.

Output response to a productivity shock



- Output increases after a positive productivity shock.
- It increases by less than in the flexible price equilibrium.
- It increases about 3 times more than in the model with a money rule.

Output response to a productivity shock II

One way to think about this is in terms of supply and demand. The production function tells us that supply and potential supply increase and will decrease over time,

$$\hat{Y}_t = \hat{A}_t + \hat{H}_t \tag{35}$$

Demand is given by the NK IS-curve:

$$\mathbb{E}_t \hat{Y}_{t+1} - \hat{Y}_t = \frac{1}{\gamma} \left[(i_t + \ln(\beta)) - \mathbb{E}_t \pi_{t+1} \right], \tag{36}$$

To mimic the shape of potential supply, we require a fall in the real interest rate. Suppose the future output gap is zero. In that case, $\pi_{t+s} = 0$ and $\Delta i_t = 0$, i.e., the real interest rate is unchanged. \Rightarrow We must have a positive output gap, i.e., demand does not rise sufficiently to match potential supply.

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The central bank responds to the output gap by decreasing the interest rate:

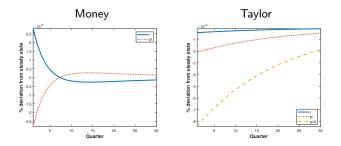
$$i_t - i^{ss} = \kappa_\pi \pi_t + \kappa_y (\hat{Y}_t - \hat{Y}_t^f) + \omega_t.$$
(37)

Hence, from the NK IS-curve,

$$\mathbb{E}_t \hat{Y}_{t+1} - \hat{Y}_t = \frac{1}{\gamma} \left[(i_t + \ln(\beta)) - \mathbb{E}_t \pi_{t+1} \right], \tag{38}$$

we see that the fall in the interest rate stimulates demand today, i.e., the central bank brings output closer to its optimal level.

Inflation response to a productivity shock



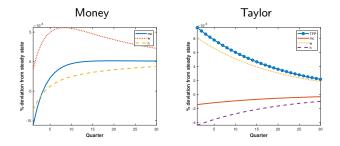
- As in the model with money demand, inflation falls.
- Again, targeted inflation falls by more than realized inflation because of the price stickiness.
- Different from before, the real interest rate now responds negatively.

As just seen, we have a negative output gap and, hence, the NK Phillips-curve implies deflation:

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}(\eta+\gamma)\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s [\hat{Y}_t - \hat{Y}_t^f]$$
(39)

In the model with money, the nominal interest rate was constant leading to a rise in the real interest rate. Now, the central bank lowers the nominal interest rate leading to a fall in the real rate. • Stability

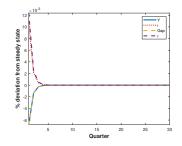
Hours response to a productivity shock



- As before, real marginal costs decrease ⇒ wages rise by less than labor productivity ⇒ hours worked fall.
- However, wage responses are stronger and hours responses weaker because the output gap is smaller:

$$\hat{m}c_t = (\eta + \gamma)[\hat{Y}_t - \hat{Y}_t^f].$$
(40)

Output response to an interest rate shock



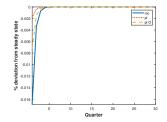
- A rise in the nominal interest rate will rise the real interest rate because of sticky prices.
- Output (the output gap) falls.
- As the error is persistent, so are responses.

From the NK IS-curve,

$$\mathbb{E}_t \hat{Y}_{t+1} - \hat{Y}_t = \frac{1}{\gamma} \left[(i_t + \ln(\beta)) - \mathbb{E}_t \pi_{t+1} \right], \tag{41}$$

we know that a rise in the real interest rate increase output growth, i.e., it depresses output today. The nominal interest rate is neutral in the flexible price equilibrium, and hence, the entire change results in a negative output gap.

Inflation response to an interest rate shock



• The fall in the output gap implies that marginal costs must fall:

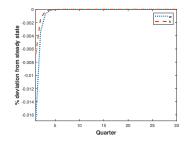
$$\hat{m}c_t = (\eta + \gamma)[\hat{Y}_t - \hat{Y}_t^f].$$
(42)

• Hence, inflation falls:

$$\pi_t = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \hat{m} c_{t+s}.$$
 (43)

• This amplifies the rise in the real interest rate.

Hours response to an interest rate shock



- Given the fall of marginal costs and no change in labor productivity
- \Rightarrow wages fall.
- \Rightarrow hours worked fall.

| | Y | С | Н | TFP | W | i | π | | | |
|--------|-----------------|-----------------|------|------|------|------|-------|--|--|--|
| | 1.61 | 1.05 | 1.0 | Data | 0.00 | 1.07 | 0.0 | | | |
| Std. % | 1.61 | 1.25 | 1.9 | 1.25 | 0.96 | 1.07 | 0.6 | | | |
| | | | - | | _ | | | | | |
| | | RBC $\eta=$ 0.5 | | | | | | | | |
| Std. % | 1.56 | 0.45 | 0.52 | 1.24 | 1.10 | 0.0 | 0.6 | | | |
| | | | | | | | | | | |
| | NKM $\eta=$ 0.5 | | | | | | | | | |
| Std. % | 0.93 | 0.93 | 0.87 | 1.24 | 1.94 | 1.07 | 0.33 | | | |
| | | | | | | | | | | |

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Correlations

| | Y | С | Н | TFP | W | π | |
|-------------------|--------|-------|-------|-------|----------------|------------------|----------|
| Data | 1 | | | | | | |
| RBC | 1 | | | | | | |
| NKM | 1 | | | | | | |
| | 0.78 | 1 | | | | | |
| С | 0.94 | 1 | | | | | |
| | 1 | 1 | | | | | |
| | 0.87 | 0.69 | 1 | | | | |
| Н | 0.92 | 0.74 | 1 | | | | |
| | 0.06 | 0.06 | 1 | | | | |
| | 0.79 | 0.71 | 0.49 | 1 | | | |
| TFP | 1 | 0.94 | 0.93 | 1 | | | |
| | 0.71 | 0.71 | -0.66 | 1 | | | |
| | 0.12 | 0.29 | -0.06 | 0.34 | 1 | | |
| W | 0.98 | 0.99 | 0.84 | 0.98 | 1 | | |
| | 0.97 | 0.97 | 0.28 | 0.54 | 1 | | |
| | 0.28 | 0.37 | 0.23 | 0.25 | 0.32 | 1 | |
| π | -0.34 | -0.23 | -0.42 | -0.34 | -0.29 | 1 | |
| | -0.23 | -0.23 | 0.96 | -0.84 | < /₽ > 0 = > · | (=) 1 = = | n 200 |
| Felix Wellschmied | (UC3M) | | NKM | | | | 40 / 47 |

- Overall, the model looks similar to the money growth rule model.
- The major gain is that the nominal interest rate is now volatile (targeted).
- Hours are no longer more volatile than output because the output gap is less volatile.
- Inflation is now countercyclical again because the output gap is less volatile.

Appendix

Determinacy of equilibrium

It is easier to work with the 3-equation model. Ignore the interest rate rule for the moment. For readability, define $\hat{X}_t = \hat{Y}_t - \hat{Y}_t^f$ and $\xi = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}(\eta + \gamma)$:

$$\mathbb{E}_t \hat{X}_{t+1} - \hat{X}_t = \frac{1}{\gamma} \left[(i_t - i^{ss}) - \mathbb{E}_t \pi_{t+1} - \tilde{r}_t^f \right]$$
(44)
$$\pi_t = \xi \hat{X}_t + \beta \mathbb{E}_t \pi_{t+1}.$$
(45)

Substituting yields:

$$\mathbb{E}_t \pi_{t+1} = \frac{\pi_t - \xi \hat{X}_t}{\beta}$$

$$\mathbb{E}_t \hat{X}_{t+1} = \hat{X}_t + \frac{1}{\gamma} \left[(i_t - i^{ss}) - \frac{\pi_t - \xi \hat{X}_t}{\beta} - \tilde{r}_t^f \right].$$
(46)
(47)

Writing the model in matrix form yields:

$$\begin{bmatrix} \mathbb{E}_t \pi_{t+1} \\ \mathbb{E}_t \hat{X}_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\xi}{\beta} \\ -\frac{1}{\beta\gamma} & 1 + \frac{\xi}{\beta\gamma} \end{bmatrix} \begin{bmatrix} \pi_t \\ \hat{X}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\gamma} \left[(i_t - i^{ss}) - \tilde{r}_t^f \right] \end{bmatrix}$$
(48)

- One can show that the matrix has one eigenvalue inside and one outside of the unit circle.
- We have two jumpers and, hence, require two eigenvalues outside of the unit circle to have a unique solution.
- With too few unstable eigenvalues we have indefinitely many solutions.
- Hence, the interest rate rule needs to assure a unique equilibrium!

$$i_t - i^{ss} = \kappa_\pi \pi_t + \kappa_y \hat{X}_t + \epsilon_t^i \tag{49}$$

Adding an interest rate rule, one can show that a unique equilibrium exists:

$$\kappa_{\pi} + \frac{(1-\beta)\kappa_{y}}{\xi} > 1.$$
(50)

- The monetary authority needs to react strongly enough to inflation and/or output deviations.
- Obviously, $\kappa_{\pi} > 1$ is sufficient.
- This implies that the real interest rate moves against inflation.
- Otherwise, rising inflation lowers the real interest rate which raises output and further raises inflation.

- Clarida, Gali, and Gertler (2000) find that the FED was not satisfying this so called Taylor principle before the appointment of Paul Volker.
- This provides a possible explanation for the very volatile and high inflation rates during that time.
- Since then, estimates suggest that the FED satisfies the Taylor principle.
- Back

- John B Taylor. "Discretion versus policy rules in practice". In: *Carnegie-Rochester conference series on public policy*. Vol. 39. Elsevier. 1993, pp. 195–214.
- [2] Richard Clarida, Jordi Gali, and Mark Gertler. "Monetary policy rules and macroeconomic stability: evidence and some theory". In: *The Quarterly journal of economics* 115.1 (2000), pp. 147–180.